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The measurement of the fundamental constants

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Some recent precision experiments that are likely to influence the accepted values of the fundamental constants are reviewed briefly: the measurement of the velocity of light, the possibility of redefining the metre in terms of the caesium time standard, developments that may allow the introduction of an atomic mass standard, the use of the Josephson effect to maintain electrical standards, and some experiments that have led to an improved precision for the fine structure constant.

THE EXPERIMENTAL MEASUREMENT OF FUNDAMENTAL CONSTANTS

Measurement of physical quantities

The measurement of any physical quantity always requires in effect an apparatus capable of comparing what is to be measured with a standard unit of the same quantity. It is important that the standard be reproduceable in different countries, that it should not drift with time or be disturbed by external influences such as temperature. The measuring apparatus acting as a comparator or null detector must have sufficient sensitivity to achieve the precision required; it should be possible to estimate the magnitudes of the known systematic errors associated with the system so that in the end the precision is limited by the random effects introduced by the standard and the measuring apparatus. The standard error quoted is a measure of the overall uncertainty in the sense that a repeat measurement is unlikely to fall outside three standard deviations from the result given.

It is the job of the laboratories such as The National Physical Laboratory in England (N.P.L.) to maintain standards of the highest precision possible so that the measuring apparatus used in the individual laboratories in universities and industry can be calibrated. Resistance and voltage, for example, can be measured to one part in 10^6, time delays of many seconds to about one microsecond and frequency to the order one part in 10^{12}. During the last twenty years there has been a significant increase in the accuracy available for many measurements as direct result of a number of technical developments and the automation possible with modern electronics.

Fundamental constants

Many of the measurements made in the physical sciences are designed to test theories: theories that predict the way some part of the solar system might develop, for example, models of superconductivity, or the way electromagnetic radiation interacts with atoms in a laser. The theory often predicts relations between directly measurable quantities such as frequency, magnetic field, temperature and the so called 'fundamental constants', the set of quantities that, at any time, combined with the appropriate theory, should be able to explain at both the microscopic and the macroscopic level the behaviour of physical systems. Some of the more obvious fundamental constants are G, the gravitational constant, c the speed of light, h Planck's constant, e the charge on the electron, m_e the mass of the electron, m_p the mass of the proton, and k Boltzmann's constant. Developments in fundamental particle physics have required the introduction of another group of fundamental constants which cannot be predicted in terms of those already mentioned, constants such as G_v the coupling constant for weak nuclear interactions, the masses of various mesons and the Weinberg angle, which relates the strengths of charged and neutral weak currents.

The number of fundamental constants in use tends to increase with time as the experiments and theories become more diverse, but there is always the hope that additional relations between the constants will emerge when new unification theories are developed. The magnetic moment of the proton, which can be measured as accurately as we can reproduce a unit of magnetic field, has to be treated as an independent constant because we have no accurate theory for the proton magnetic moment in terms of other constants. The magnetic moment of the electron, on the other hand, is accurately predicted in terms of e, h, m_e and c by Dirac's theory of the electron plus the theory of quantum electrodynamics and does not have to be treated as a fundamental constant.

The least squares adjustment of fundamental constants

Many of the precision measurements made on physical systems can be related to the magnitudes of the SI units and the relevent fundamental constants when well tested theories are available. It is then possible to combine selected accurate results together with their individual standard errors in a least squares adjustment procedure that changes the values given in the constants until a set of 'best values' emerge, values which are consistent with the experimental results and the theories. This process was last carried out by Cohen (Cohen & Taylor 1973) and since then many research groups and specialist metrologists in standards laboratories have been developing a new generation of measurement techniques to provide input data for the evaluation of a new set of 'best values' by the international Committee on Data for Science and Technology (CODATA) task group on fundamental physical constants who expect to report their results at the end of 1983. Each analysis of this type yields a number of discrepancies which have to be resolved and in this process we improve our understanding of the physical world in the sense that the theories combined with the values of the constants provide a more consistent description of the experimental results.

We see, then, that the experimental determination of the fundamental constants involves the careful choice of accurate experiments which will influence favourably the least squares adjustment, for we are unable to make a direct precise measurement of a fundamental constant such as Planck's constant h or the proton mass m_p. The remainder of this paper will be concerned with some recent measurements of this type leaving others which involve the gravitational constant G, the constants associated with high energy physics and some aspects of cosmology to other papers in the conference.

Time, length and the velocity of light

More than 30 measurements of the velocity of light c, the constant that occurs so often in theories of relativity and electromagnetism, have been made since the earliest recorded experiment by Roemer in 1675. One of the more accurate experiments by Michelson in 1927 involved the direct measurement of the time taken for light pulses to travel a distance of 44 miles† and yielded an accuracy of the order $67/10^6$. Most of the measurements since 1950 have made use of the fact that c is equal to the product of frequency and wavelength for an electromagnetic wave in a

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† 1 mile = 1.60934 km.

vacuum. The most recent experiments by Evenson *et al.* in 1973 at the Bureau of Standards (U.S.A.) and Jolliffe *et al.* (1974) at the National Physical Laboratory (England) gave a much improved accuracy of $0.003/10^6$ because the measurements of both frequency and wavelength in vacuo were made simultaneously against the primary SI standards.

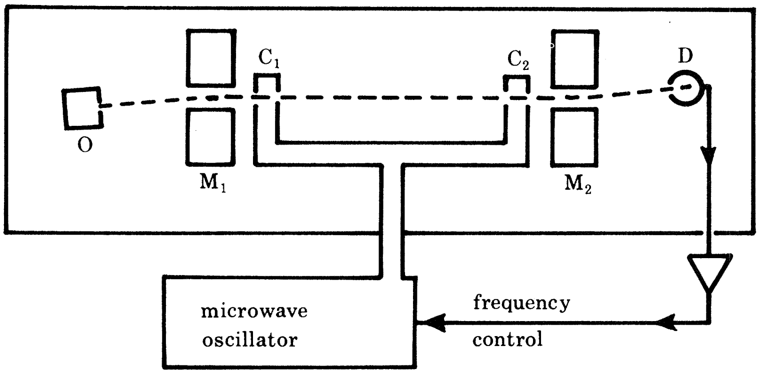

FIGURE 1. The caesium atomic clock. Caesium atoms leaving the oven O reach the detector D after passing through the inhomogeneous field focusing magnets M_1 and M_2 if they enter the microwave cavity C_1 in one hyperfine level W_1 and leave the cavity C_2 in the other level W_2. Resonance occurs when the microwave frequency f satisfies $hf = (W_1 - W_2)$ and the linewidth is $1/2T$ where T is the transit time from C_1 to C_2.

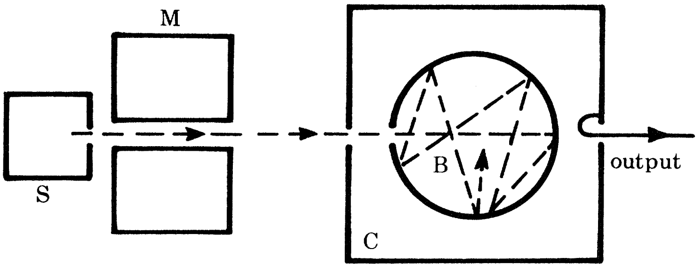

FIGURE 2. The hydrogen maser. Hydrogen atoms leaving the discharge source S in the upper hyperfine level W_1 are focused by the 4 or 6 pole magnet M into the coated bulb B in a microwave cavity C. Continuous oscillation at the hyperfine transition frequency $f = (W_1 - W_2)/h$ occurs when the beam intensity is increased until the induced downward transition rate to the lower level W_2 due to the microwave field already in the cavity is sufficient to make up the losses.

The SI primary standard of frequency and time is the atomic beam caesium clock, an apparatus in which a microwave oscillator is locked to the frequency $f = (W_1 - W_2)/h$ which maximizes the transition rate between two hyperfine levels W_1 and W_2 in the ground state of atomic ^{133}Cs moving without collisions in a vacuum. The second in SI units is formally defined as 9 192 631 770 periods of such an oscillator running under well defined conditions, a standard which has the amazing long term stability of 1 part in 10^{14} and an accuracy or reproduceability of 1 part in 10^{13}.

Well designed hydrogen masers oscillate at the microwave frequency associated with transitions between the ground state hyperfine levels of hydrogen atoms stored in a container. They have a better stability than the caesium clock over a period of a week to 10 days, about 1 part in 10^{15}, and they are more easily flown in satellites to carry out tests of relativity. They are not so good, however, as absolute time standards because the atoms collide with the walls of the container and the resultant reproduceability is only 1 part in 10^{12}.

The metre, the unit of length originally defined in terms of two scratch marks on the platinum–iridium bar kept at the Bureau International des Poids et Measures (B.I.P.M.) near Paris, was redefined in 1960 as 1 650 763.73 vacuum wavelengths of one of the orange–red lines in the spectrum of a ^{83}Kr lamp running under well defined conditions. The spacing between a pair of optically flat coated plates can be measured interferometrically in SI units with an accuracy of the order 1 part in 10^8, the limitation being mainly the linewidth and reproduceability of the krypton light source. The krypton standard for the metre was defined before the invention of the laser and the possibility of stabilized light sources in the infrared and visible with a short term stability approaching 1 part in 10^{12}, so we have every reason to expect a redefinition of the metre in the near future.

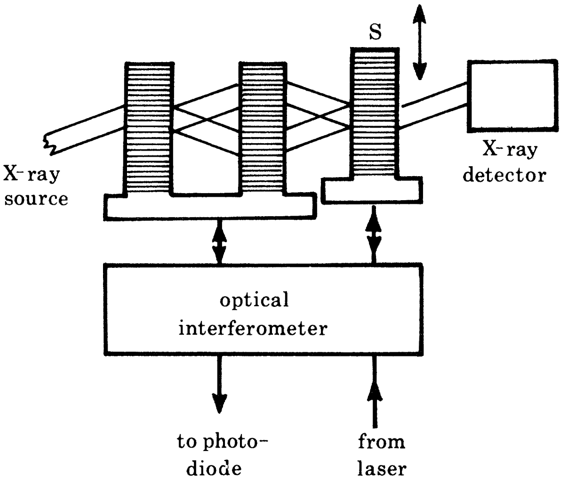

FIGURE 3. Measurement of a lattice plane separation in silicon. Movement of the silicon crystal S in the direction shown produces a pattern with period *d* equal to the lattice spacing from the X-ray detector and a pattern with a period equal to half an optical wavelength from the optical detector. Careful analysis of the patterns recorded simultaneously as the crystal moves allows a direct estimate of *d* in terms of the wavelength of the light.

The technique used at N.P.L. to measure the speed of light involved a direct comparison of the wavelengths of the standard line from the krypton lamp and radiation at a similar frequency of 5×10^{14} Hz obtained by mixing light from several highly stabilized lasers. The laser frequencies were measured by complicated beating techniques in terms of the atomic frequency standard at about 10^{10} Hz and the resultant speed of light deduced from the frequency wavelength product had an error limited mainly by the linewidth of the krypton lamp.

Because of the uncertainty in any measurement of a length in terms of the krypton standard it is possible that the next meeting of the Conference General des Poids et Measures in 1983 may define the metre in terms of the velocity of light as the distance travelled in a vacuum in the time 1/2 997 924.58 s measured in terms of the caesium clock. This definition will keep the magnitudes of the length and time standards at their present ‘best values’ and allow frequency stabilized lasers to be used as local transfer standards to obtain more precise length measurements.

Mass

The present mass standard is related to the international prototype of the kilogram kept at B.I.P.M. in France and local standards calibrated by direct weighing to about $0.01/10^6$ are kept in the national standards laboratories. Although atomic masses can be compared in mass spectrometers with a precision approaching $0.01/10^6$ an atomic mass standard cannot be used at present because the Avogadro number, which gives the number of atoms in a mass of material equal to the atomic number in grams, is not known with sufficient precision. This situation may change in the near future since Becker *et al.* (1981) showed that it is possible to make an accurate measurement of the lattice spacing in a pure perfect crystal of ^{26}Si by using X-ray diffraction and optical interference to measure the number of lattice spacings in a half wavelength of light. This will allow the mass standard to be redefined in terms of the mass of a silicon crystal of known dimensions and leave us with standards of mass, length and time in terms of natural atomic systems.

Electrical standards

Electronic techniques play a crucial role in any precision measurement so it is important that the accuracy of the electrical standards continue to be improved beyond the $1/10^6$ level typically available in well equipped laboratories. A very significant advance has been made in this direction in the last 20 years by the realization that the Josephson effect (Josephson 1962) can be used to define the volt in terms of the unit of time. It is found that a superconducting current passes by tunnelling across a gap as small as 1 nm separating two superconductors when the voltage V across the gap and the frequency f of microwave radiation applied are given by $2eV = nhf$, where n is an integer and h is Planck's constant. The ratio $2e/h$ measured in this way is found to be independent of the materials used to form the junction to better than $0.001/10^6$ (Gallop 1928) and most standards laboratories are now using the technique to maintain the volt in terms of $2e/h$ with a precision of about $0.01/10^6$. Details of the way this is done are described in recent reviews by Petley (1980) and Gallop (1982).

The accurate measurement of magnetic field is usually carried out by a magnetic resonance technique involving the protons in water or some other system with magnetic moment μ and spin J. Transitions between the states in the field B are detected when the frequency f of an additional oscillating field is given by $hf = \mu B/J$, so it is important that the relevent nuclear and atomic magnetic moments are available with high precision. Measurements such as the ratio of the proton resonance frequency to the proton cyclotron circulation frequency in a given field as well as the magnitude of the proton moment in hertz per tesla provide important data for the least squares adjustment of the fundamental constants and also lead to an accurate value for the fine structure constant α.

The fine structure constant

The fine structure constant $\alpha = e^2\mu_0 c/2h$ is a particularly important dimensionless combination of fundamental constants with the value $1/137.035\,965$ and uncertainty $0.09/10^6$. The fine structure splitting in atoms and many quantum electrodynamic expressions for quantities such as the electron g_e factor and the proton gyromagnetic ratio can be expressed as a power series in α. The electron and positron g factors have recently been measured (Schwinberg *et al.* 1981) with an accuracy of 4 parts in 10^{11} by studying the behaviour of single electrons and positrons stored in an electromagnetic trap. They are probably the most accurate measurements of any property of a fundamental particle made so far but the value of α^{-1} deduced from the results is $0.33 \pm 0.14/10^6$ greater than that derived from an accurate measurement of the proton gyromagnetic ratio (Williams & Olsen 1979), a discrepancy which may imply a need for a higher power of α in the expression for g_e. The fine structure constant has also been deduced by Tsui *et al.* (1982) with an accuracy of $0.09/10^6$ by measuring the quantized Hall resistance of a gallium arsenide hetrostructure in which a two dimensional electron gas is formed and the result is in good agreement with the proton gyromagnetic ratio value.

CONCLUSIONS

The last 20 years have seen a steady change over from standards like the standard cell and the quartz clock, which required very careful control during manufacture and an extremely stable environment to obtain good precision, to standards based on well understood quantum systems. Already the second depends on the atomic caesium clock and the volt is maintained by observing Josephson junction voltage steps. Before long it is likely that the metre will be defined in terms of an adopted value for the speed of light and the caesium second, the kilogram will be related to an improved Avogadro number based on the lattice spacing in a crystal, and the ohm may well be defined in terms of quantized Hall resistance. It will then be much easier to maintain precise standards in different research laboratories around the world and this in turn will have a beneficial effect on the tests of basic theories and the determination of the fundamental constants, developments that must necessarily proceed together if the least squares adjustment technique is used to determine 'best values' for the constants. We cannot predict at the moment whether the so called 'fundamental constants' will change with time as the Universe expands but there is little doubt that interesting new discoveries and theories will emerge as the precision improves.

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